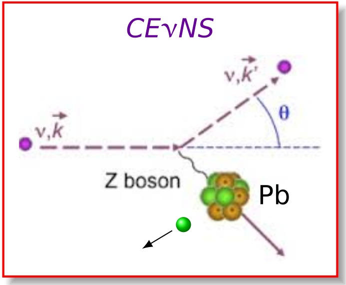
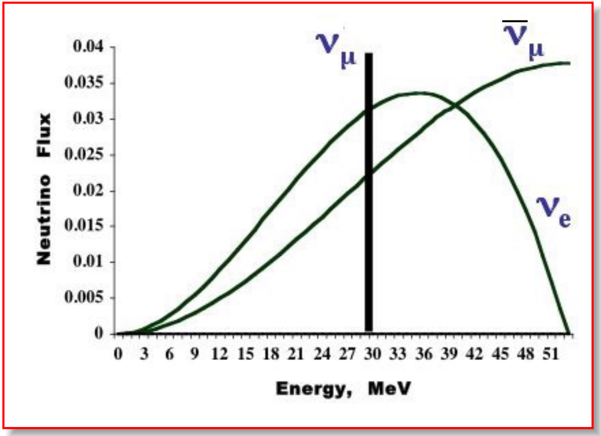
Inelastic Neutrino-Nucleus Scattering

J. Engel

Jan 12, 2015



Spectrum from Stopped Pions



Want to

1. Calculate neutron background for CEνNs expt.
2. Use CEνNs expt. to test calculations of supernova-ν cross sections.

What Needs to Be Done

In short:

1. Calculate matrix elements of operators like

$$j_l(qR)Y^l_m(\Omega) \qquad j_l(qr)Y^l_m(\Omega)\boldsymbol{\sigma}$$

$$j_l(qR)Y^l_m(\Omega)\boldsymbol{\nabla} \qquad j_l(qr)Y^l_m(\Omega)\boldsymbol{\nabla}\boldsymbol{\sigma}$$

   for all kinematically allowed q to all energetically accessible nuclear excited states, both for charge conserving and charge-changing operators.
2. Then calculate emission of neutron, protons, photons, etc.

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I'll focus on this.

The emission of particles has always been treated statistically, e.g. through Hauser-Feshbach theory. Improvement would be helpful but may be difficult.

Options for Calculating Excitation

1. Shell model, perhaps supplemented by collective treatment of negative-parity giant resonances.
2. Ab initio calculation. Coupled-clusters theory has already been applied to ^{16}O and could be extended to spherical nuclei at least as heavy as Ca or Ni.
3. Some variant of RPA, or improved RPA.

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3. Some variant of RPA, or improved RPA.

For neutrinos with 10's of MeV the last option is probably the best in heavy nuclei (though you should feel free to challenge this assertion).

Varieties and Extensions of RPA

- Like-particle for neutral current, charge-changing for charged current.
- Include pairing (QRPA).
- Include deformation.

  Deformation not necessary for Pb, Fe, Ar, O.

- Include continuum, approximately (e.g. through box boundary conditions) or exactly.
- Include multi-phonon states (e.g. second RPA).
- ⋮

Step 1 in QRPA: Pairing and Quasiparticles

Generalize mean field theory to include pairing correlations. The optimal paired state

$$|HFB\rangle \equiv \mathcal{N} e^{\frac{v_a}{u_a} a_a^\dagger a_{\bar{a}}^\dagger} |HF\rangle$$

(in the "canonical basis") can be represented as a vacuum of *quasiparticles*:

$$\alpha_a |HFB\rangle = 0 \,, \qquad \alpha_a \equiv u_a a_a - v_a a_{\bar{a}}^\dagger \,.$$

“Explicit-State” Formulation of QRPA

$$|\nu\rangle = Q^\dagger_\nu |0\rangle \qquad\qquad Q_\nu |0\rangle = 0$$

in "boson" approximation $[Q_\mu, Q^\dagger_\nu] = \delta_{\mu,\nu}$.

Like-particle QRPA:

$$Q^\dagger_\nu = \sum_{pp'} X^\nu_{pp'} \alpha^\dagger_p \alpha^\dagger_{p'} + \sum_{nn'} X^\nu_{nn'} \alpha^\dagger_n \alpha^\dagger_{n'} - \sum_{pp'} Y^\nu_{pp'} \alpha_p \alpha_{p'} - \sum_{nn'} Y^\nu_{nn'} \alpha_n \alpha_{n'}$$

Charge-changing QRPA:

$$Q^\dagger_\nu = \sum_{pn} X^\nu_{pn} \alpha^\dagger_p \alpha^\dagger_n - Y_{pn} \alpha_p \alpha_n$$

There are a million other ways of thinking about RPA: linear response of mean-field, ring-diagram Green’s-function sum, coherent-state expansion, etc.

Our Framework: Skyrme Energy Density Functional

Schematicized a Bit — Isoscalar Densities Only

Semilocal density functional; exact in principle with enough terms in gradient expansion.

$$\mathcal{E} = \int d\mathbf{r} \left(\mathcal{H}^{\text{even}}(\mathbf{r}) + \mathcal{H}^{\text{odd}}(\mathbf{r})\right) ,$$

$$\mathcal{H}^{\text{even}}(\mathbf{r}) \equiv C^{\rho}\rho(\mathbf{r})^2 + C^{\Delta\rho}\rho(\mathbf{r})\boldsymbol{\nabla}^2\rho(\mathbf{r}) + C^{\tau}\rho(\mathbf{r})\tau(\mathbf{r}) + C^{J}\mathbb{J}(\mathbf{r})^2 + C^{\rho\nabla J}\rho(\mathbf{r})\boldsymbol{\nabla}\cdot\mathbf{J}(\mathbf{r})$$

$$\mathcal{H}^{\text{odd}}(\mathbf{r}) \equiv C^{s}\mathbf{s}(\mathbf{r})^2 + C^{\Delta s}\mathbf{s}(\mathbf{r})\cdot\boldsymbol{\nabla}^2\mathbf{s}(\mathbf{r}) + C^{j}\mathbf{j}(\mathbf{r})^2 + C^{T}\mathbf{s}(\mathbf{r})\cdot\mathbf{T}(\mathbf{r}) + C^{s\nabla j}\mathbf{s}(\mathbf{r})\cdot\boldsymbol{\nabla}\times\mathbf{j}(\mathbf{r})$$
$$+ C^{F}\mathbf{s}(\mathbf{r})\cdot\mathbf{F}(\mathbf{r}) + C^{\nabla s}\left(\boldsymbol{\nabla}\cdot\mathbf{s}(\mathbf{r})\right)^2$$

$$\rho(\mathbf{r}) = \sum_{i\leq F,s}|\phi_i(\mathbf{r},s)|^2 \qquad \tau(\mathbf{r}) = \sum_{i\leq F,s}|\nabla\phi_i(\mathbf{r},s)|^2$$

$$\mathbf{J}(\mathbf{r}) = -\frac{i}{2}\sum_{i\leq F,s,s'}\phi_i^*(\mathbf{r},s)\overleftrightarrow{\boldsymbol{\nabla}}\phi_i(\mathbf{r},s')\times\boldsymbol{\sigma}_{ss'} \qquad \mathbf{s}(\mathbf{r}) = \sum_{i\leq F,s,s'}\phi_i^*(\mathbf{r},s)\boldsymbol{\sigma}_{s,s'}\phi_i(\mathbf{r},s')$$

$$\mathbf{T}(\mathbf{r}) = \sum_{i\leq F,s,s'}\boldsymbol{\nabla}\phi_i^*(\mathbf{r},s)\cdot\boldsymbol{\nabla}\phi_i(\mathbf{r},s')\boldsymbol{\sigma}_{s,s'} \qquad \mathbf{j}(\mathbf{r}) = -\frac{1}{2i}\sum_{i\leq F,s}\phi_i^*(\mathbf{r},s)\overleftrightarrow{\boldsymbol{\nabla}}\phi_i(\mathbf{r},s)$$

$\mathbf{F}(\mathbf{r})$ = tensor version of $\mathbf{T}(\mathbf{r})$ $\qquad$ $\mathbb{J}(\mathbf{r})$ = tensor version of $\mathbf{J}(\mathbf{r})$

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$$+ C^{F}\mathbf{s}(\mathbf{r})\cdot\mathbf{F}(\mathbf{r}) + C^{\nabla s}\left(\boldsymbol{\nabla}\cdot\mathbf{s}(\mathbf{r})\right)^2$$

RPA in this picture is adiabatic approximation to exact linear response. Two-body RPA interaction: $V^{\mathrm{ph}}_{acbd} = \dfrac{\delta^2 E[\rho]}{\delta\rho_{ba}\delta\rho_{dc}}$,

$i \le F,s,s'$ $i \le F,s,s'$

$$\mathbf{T}(\mathbf{r}) = \sum_{i\le F,s,s'} \boldsymbol{\nabla}\phi_i^*(\mathbf{r},s)\cdot\boldsymbol{\nabla}\phi_i(\mathbf{r},s')\boldsymbol{\sigma}_{s,s'} \qquad \mathbf{j}(\mathbf{r}) = -\frac{1}{2i}\sum_{i\le F,s}\phi_i^*(\mathbf{r},s)\overleftrightarrow{\boldsymbol{\nabla}}\phi_i(\mathbf{r},s)$$

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This part completely unconstrained by even-even ground states. Need RPA even to fix functional.

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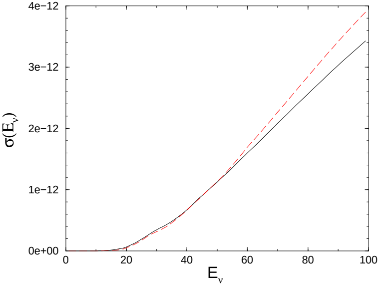
Old Results: Neutrino Scattering on ^{208}Pb (Spherical)

With Gail, Cristina Volpe

SIII ——

SkO+ - - -

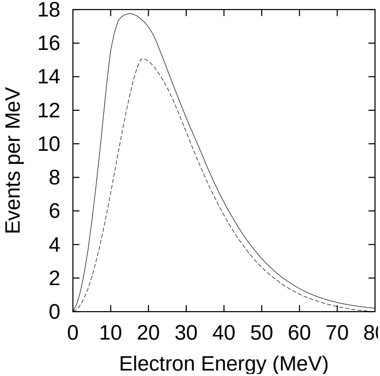


Charged current cross section for ν_e's on ^{208}Pb

Not much sensitivity to functional in total cross section. But may be more in quantities like cross section for 1-neutron emission.

One-neutron emission ——

Two-neutron emission - - -



Electron-energy spectra for supernova ν_e's on ^{208}Pb

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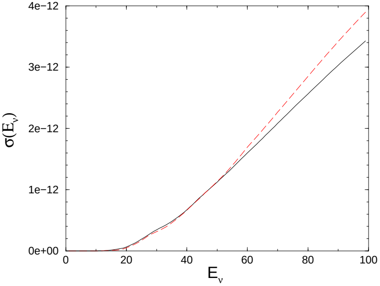

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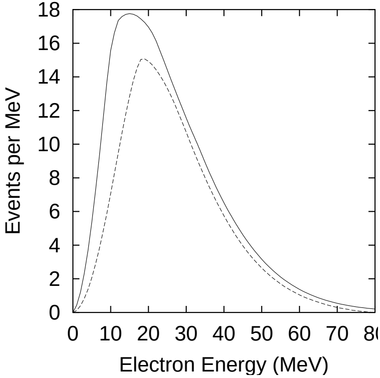


Electron-energy spectra for supernova … ^{208}…

Same codes can be used for other even Pb, Fe, O isotopes.

Finite-Amplitude Method (Deformed)

One solves for QRPA linear response directly through iteration, without ever explicitly constructing QRPA Hamiltonian matrix.

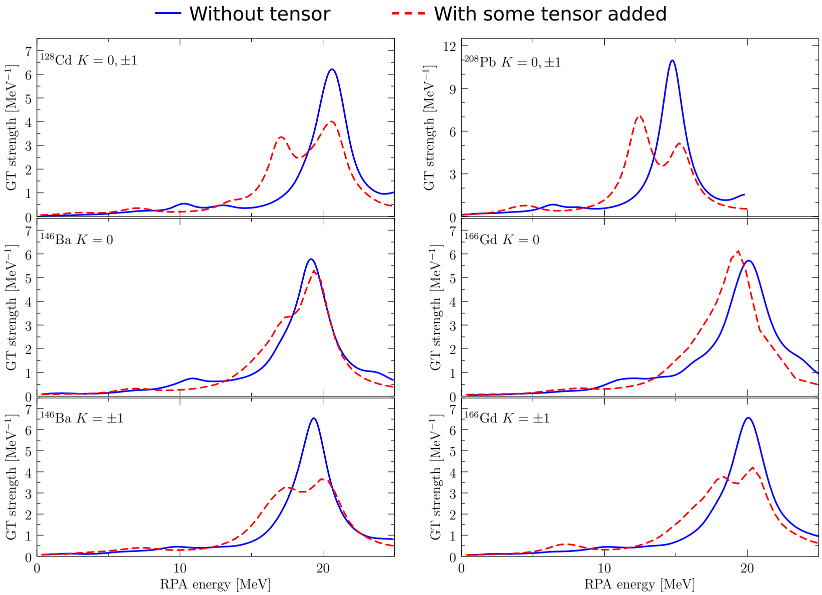
Yields orders-of-magnitude speed up, making full Skyrme QRPA calculations in very large spaces and easy.

Allows use of Gamow-Teller resonances, spin-dipole resonances, beta-decay lifetimes, etc. from all over the periodic chart to fit the time-odd functional.

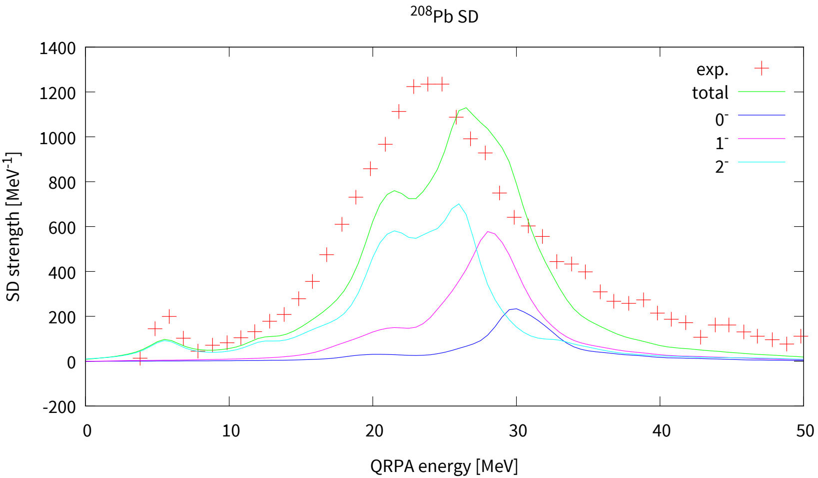
We're in the process of doing that fitting now. Should lead to better Skyrme-based neutrino cross sections than we've ever had.

Modifying the Time Odd Functional

Ex: Tensor term and Gamow-Teller Strength Distributions (M. Mustonen)



Spin-Dipole Resonances



With *SkO*+

Can Also Include Electron-Scattering Data in Fit

Serious work to reproduce related stuff should let you do better than a factor of two. How much better? Dunno. But we should also be able to quantify error by looking at correlations between ν cross sections and other quantities.

Final Development: Second RPA

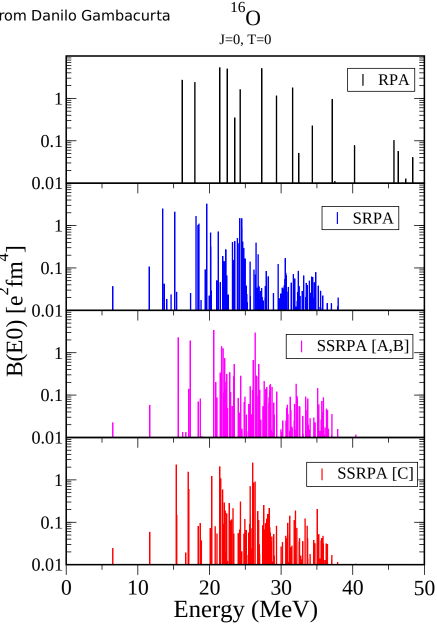
Goes beyond adiabatic response (Skyrme RPA) in a way that includes 2p-2h (two-phonon) excitations. In, e.g., charge-changing channel:

$$Q_\nu^\dagger = \sum_{pn} X_{pn}^\nu \alpha_p^\dagger \alpha_n^\dagger - Y_{pn}^\nu \alpha_p \alpha_n + \mathcal{X}_{pnp'n'}^\nu \alpha_p^\dagger \alpha_n^\dagger \alpha_{p'}^\dagger \alpha_{n'}^\dagger - \mathcal{Y}_{pnp'n'}^\nu \alpha_p \alpha_n \alpha_{p'} \alpha_{n'}$$

Can’t generalize finite-amplitude method and so we have to construct and diagonalize huge Hamiltonian matrix.

Second-RPA Work in Progress

From Danilo Gambacurta



Unadorned version doesn't play well with DFT because it adds ground-state correlation energy already accounted for by functional. Needs to be modified so that response at $\omega = 0$ is RPA response (which is exact at $\omega = 0$ if functional is exact).

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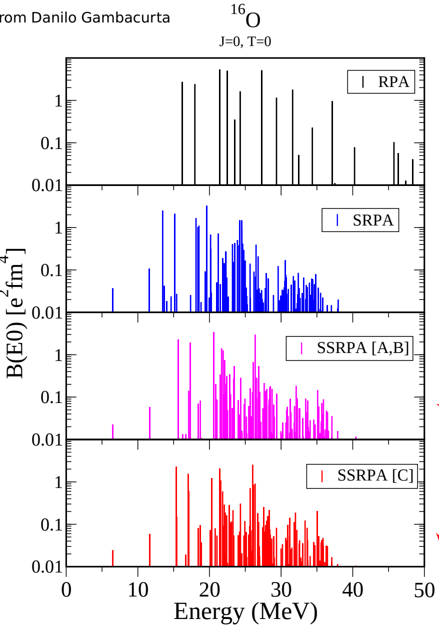

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These last two figures are with this modification in different approximations.

What About the NINs?

Excitation cross sections will need to be folded with calculations of neutron emission probabilities and propagation of neutrons into detector.

We should recruit some astro guys. . .

The End

Thanks for your kind attention.